AD-A057 694

AD-A057 694

TECHNICAL REPORT ARBRL-TR-02076

THE EVALUATION, MANIPULATION AND IDENTIFICATION OF NONDIMENSIONAL NUMBERS

Morton A. Hirschberg

June 1978

US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
BALLISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MARYLAND

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Date Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER TECHNICAL REPORT ARBRL-TR-02076	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) The Evaluation, Manipulation and Identification of Nondimensional Numbers		5. TYPE OF REPORT & PERIOD COVERED
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Morton A. Hirschberg		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS US Army Ballistic Research Laboratory ATTN: DRDAR-BLB Aberdeen Proving Ground, MD 21005		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 1T161101A91A
11. CONTROLLING OFFICE NAME AND ADDRESS US Army Armament Research & Development Command Ballistic Research Laboratory ATTN: DRDAR-BL Aberdeen Proving Ground, MD 21005		12. REPORT DATE JUNE 1978
		13. NUMBER OF PAGES 51
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

DTIC QUALITY INSPECTED 3

19. KEY WORDS (Continue on reverse side if necessary and identify by block number) fek

Buckingham Pi Theorem	Dimensionless parameters	Pi terms
Computer analysis	Formula manipulation	Similitude
Computer modeling	Nondimensional numbers	Symbol manipulation
Dimensional analysis	Nondimensional parameters	SYMBOLANG
Dimensionless numbers	Symbol manipulation	Symbolic equations
		Theory of dimensions

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report describes a computer program which evaluates, manipulates and identifies nondimensional numbers (pi terms, dimensionless numbers or factors) generated using the Buckingham Pi Theorem. Computations are performed using SYMBOLANG, an algebraic (symbol) manipulation package (solutions are kept in symbolic form). Manipulation occurs in two ways. First, many allowable combinations of pi terms (up to 720) are generated by changing the ordering of the dimensional equations which are solved. Second, a small group of operators (squaring,

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

square-rooting, cube-rooting, reciprocating, and division by both the minimum and maximum exponent) are applied to each solution (these too are solutions and also kept in symbolic form). Numerical substitution into the symbolic forms produce scalars (evaluated solutions) for subsequent use (plotting, regression, factor analysis, etc.). Finally, every symbolic solution is compared with well-known nondimensional numbers (e.g., Reynold's, Weber's, etc.). If a match is found, this information is displayed.

TABLE OF CONTENTS

		Page
I.	INTRODUCTION	5
II.	THE BUCKINGHAM PI THEOREM	6
III.	SYMBOLANG	9
IV.	THE PROGRAM	9
V.	DISCUSSION	10
	ACKNOWLEDGEMENT	11
	REFERENCES	12
	APPENDIX I - Program Computer Listings	13
	APPENDIX II - Sample Program Inputs	37
	APPENDIX III - Input Description and Format	41
	APPENDIX IV - Sample Program Outputs	45
	DISTRIBUTION LIST	49

"It is out of the question to formulate and carry out experiments nowadays without making use of similarity and dimensionality concepts."[1]

I. INTRODUCTION

One measure of the importance of a scientific area is the number of papers written in that area. Dimensional analysis has generated over 600 publications,[2] however, only a handful of these papers deal with computerized dimensionality.[3,4,5,6,7]

There are several reasons why so little computer work has been done in dimensional analysis. First, much work has been devoted to methodologies and specific solutions in highly specialized areas. Once this work has been done, a computer is not necessary. Second and more important is that computer codes have been inflexible. Only one solution is usually generated for a given ordering of dimensional equations. Therefore, fruitful and/or well recognized solutions might not be generated. This, just because of ordering. Next, identifiable and well recognized solutions are not identified. The user does needless computational work when this occurs. Finally, with the exception of the work of Cohen and Ferrari (Reference 6), solutions are not usually numerically evaluated. Therefore, a great deal of computational work must be performed before suitable data is available for plotting, regression, clustering, or factor analysis.

---

[1]Sedov, L. I. Similarity and Dimensional Methods in Mechanics. New York, Academic Press, 1959.

[2]Happ, W. W., Private Communication, January 1976.

[3]Happ, W. W., "Computer-Oriented Procedures for Dimensional Analysis." Journal of Applied Physics, 38, 3918-3926, September 1967.

[4]Sloan, A. D. and Happ, W. W., "Computer Program for Dimensional Analysis." Electronics Research Center, Cambridge, Mass., NASA TN D-5165, April 1969.

[5]Chen, W. K. "Algebric Theory of Dimensional Analysis." Journal of the Franklin Institute, 292, 403-422, December 1971.

[6]Cohen, J. and Ferrari, J. O., "A Conversational Language for Solving Problems in Dimensional Analysis." Computer Methods in Applied Mechanics and Engineering, 5, 53-67, January 1975.

[7]Hirschberg, M.A., "A Computer Solution of the Buckingham Pi Theorem Using SYMBOLANG, A Symbol Manipulation Language," USA Ballistic Research Laboratories Report No. 1824, August 1975. (AD #A016901)

The program described in this paper overcomes most of the named deficiencies, that is, those of inflexibility. Computer solutions are generated using the Buckingham Pi Theorem (Section II). Computations are performing using SYMBOLANG (Section III), an algebraic (symbol) manipulation package. Solutions to dimensional equations are kept in symbolic form. In addition, solutions are manipulated in two ways. First, many allowable combinations of pi terms (up to 720) can be generated by the program automatically changing the order of the dimensional equations. Second, a small group of operators (squaring, cubing, square-rooting, cube-rooting, reciprocating, and division by both the minimum and maximum exponent) are applied to each generated solution. Various transformations applied to solutions produce new solutions. Products of solutions also produce new solutions. Provision has been made for numerical substitution into the symbolic forms producing scalars (evaluated solutions) for subsequent use. In addition, every symbolic solution is compared with well-known nondimensional numbers (e.g., Reynold's, Weber's, etc.). Well-known numbers are selected from the Land Table[8]. When a match between a symbolic and a Land number is found, the name of the Land number is printed for all such numbers which match (i.e., there may be more than one name printed for a particular solution).

II. THE BUCKINGHAM PI THEOREM

The Buckingham Pi Theorem[9] summarizes the entire theory of dimensional analysis.[10] The result of a dimensional analysis is the reduction of the number of variables in a problem. Application of the Pi theorem itself provides the method of solution of a set of dimensional equations. Simply stated, the pi theorem asserts:

> If there are n variables involving N fundamental units, these may be combined to form n-N dimensionless parameters each involving N + 1 variables.

The usual method of applying the Pi theorem is for one to write equations describing the physical system one is interested in in terms of a set of fundamental units (force, length, time, angle, or mass, length time, etc.). The equations are then systematically exponentiated and multiplied together (hence "Pi" theorem for the mathematical symbol for multiplication (π). The resulting exponentials form an N by N set of

---

[8] Land, N. S., "A Compilation of Nondimensional Numbers," Washington, DC, US Government Printing Office, NASA SP-274, 1972.

[9] Buckingham, E., "On Physically Similar Systems: Illustrations of the Use of Dimensional Equations," Physical Review, 2, 345-376, 1914.

[10] Langhaar, H. L., Dimensional Analysis and Theory of Models, New York, Wiley, 1951.

linear equations whose solution is applied back to the variables of the problem (see the example below). One can see that it is tedious to work a problem involving many variables by hand. In fact, there is no good indicator of just how many variables one should include in a problem (this is true even for a computerized solution).

The following example (taken from Housner and Hudson[11]) used throughout the remainder of the paper illustrates the mechanisms of the Buckingham Pi Theorem.

> Consider a drag force (F) acting upon a body moving through a fluid. Assume a constant velocity (V) through the fluid of density (ρ) and viscosity (μ). If the analysis is applied to bodies of a specific shape, the cross-sectional area (A) may be used as a measure of the body's size.

The following variables and fundamental units enter into the problem:

Variable		Fundamental Units
F	=	F
μ	=	$FL^{-2}T$
A	=	L^2
ρ	=	$FL^{-4}T^2$
V	=	LT^{-1}

where F = Force, L = Length and T = Time,

According to the Pi theorem, two terms can be formed from the five equations (each equation is expressed in terms of the three fundamental units). The two solutions will each contain four of the variables. The pi terms formed with this ordering are:

$$\pi_1 = FA^{\alpha}\rho^{\beta}V^{\gamma} = F^{1+\beta}L^{2\alpha-4\beta+\gamma}T^{2\beta-\gamma}$$

and

$$\pi_2 = \mu A^{\alpha}\rho^{\beta}V^{\gamma} = F^{1+\beta}L^{-2+2\alpha-4\beta+\gamma}T^{1+2\beta-\gamma}$$

Solving these equations we find: π_1 has the solution $\alpha = -1$, $\beta = -1$, $\gamma = -2$; π_2 has the solution $\alpha = -1/2$, $\beta = -1$, $\gamma = -1$; so the resulting dimensionless pi terms are:

[11] Housner, C. W. and Hudson, D. E., Applied Mechanics Dynamics. New York, van Nostrand, 1950.

$$\pi_1 = \frac{F}{A\rho V^2} \qquad \text{and} \qquad \pi_2 = \frac{\mu}{A^{\frac{1}{2}}\rho V}$$

π_1 is a pressure coefficient and π_2 is the reciprocal of the Reynold's number[10]. One can see how the equations are systematically selected and visualize the ease with which such an algorithm can be programmed.

The above represent one set of solutions. In this particular case, eight other solutions are possible depending upon the reordering of the five basic equations. The full set of solutions is listed below:

(1)	$\frac{F}{A\rho V^2}$	(6)	$\frac{A\rho V^2}{F}$
(2)	$\frac{\mu}{A^{\frac{1}{2}}\rho V}$	(7)	$\frac{VA^{\frac{1}{2}}\rho^{\frac{1}{2}}}{F^{\frac{1}{2}}}$
(3)	$\frac{A\mu^2 V^2}{F^2}$	(8)	$\frac{\mu}{F^{\frac{1}{2}}\rho^{\frac{1}{2}}}(A^0)$
(4)	$\frac{\rho F(V^0)}{\mu^2}$	(9)	$\frac{V\mu A^{\frac{1}{2}}}{F}$
(5)	$\frac{\mu}{\rho^{\frac{1}{2}}F^{\frac{1}{2}}}(V^0)$	(10)	$\frac{\rho F(A^0)}{\mu^2}$

In this example, only solutions (1) and (2) are fruitful (contain information). The other eight solutions can all be derived from solutions (1) and (2). We must note, however, that solutions (4) and (10) are minimal in the sense that they (a) contain the fewest number variables and (b) the sum of their (integer) exponents is a minimum (see References 4 and 5 for a further discussion of minimal solutions).

III. SYMBOLANG

SYMBOLANG[12,13], a high-level FORTRAN language for algebraic (symbol) manipulation, was used to form the symbolic products (pi terms) of the fundamental units. This application was well suited for SYMBOLANG. Not only were solutions generated, but one was able to see the development of the pi terms as the products were being formed. In addition, the final equation was also displayed. SYMBOLANG is of pedagogical value in demonstrating the mechanisms of the Buckingham Pi Theorem.

IV. THE PROGRAM

Listings of the routines* described in this section appear in Appendix I. The many SLIP* and SYMBOLANG* routines are not included; however, they are available from the authors of Reference 12.

The name of the main program is BUCKY. BUCKY initializes the SLIP-SYMBOLANG working storage area, then reads and displays the inputs (sample inputs appear in Appendix II while an explanation of the inputs and formats required for them appears in Appendix III). BUCKY next calculates the number of combinations (orderings) possible based upon the number of equations input; however, only 720 permutations are allowed. BUCKY next breaks the equations into two pieces; the ones to the left of the equal sign are variables and those to the right of the equal sign are fundamental units. Now, the Pi theorem algorithm is invoked. This is where the exponentiation and multiplication of fundamental units occur. The result of this step is the system of equations which is solved using a matrix inverse routine. The solution of the linear equations is performed in subroutine BUCKSV. Next, the appropriate numerical operator (squaring, etc.) is established and a numerical substitution made. A scalar is produced.

* Variable length calls and calls to functions have been modified to run on the BRLESC II Computer.

---

[12]Findler, N. V., Pfaltz, J. L., and Bernstein, H. J., Four High-Level Extensions of FORTRAN IV; SLIP, AMPPL-II, TREETRAN, SYMBOLANG. New York, Spartan, 305-387,1972.

[13]Hirschberg, M. A., "SYMBOLANG - A SLIP Extension for Algebraic Manipulation," USA Ballistic Research Laboratory Report No. 1749, November 1974. (AD #A003190)

Evaluation is performed in subroutine EVALP. EVALP is an inelegant routine whose main virtue lies in producing the correct scalar value. Up to five values may be input for each of 24 variables (a considerable number for a dimensional analysis). Reciprocating is also performed in EVALP.

After evaluation, the exponents of the symbolic solution are passed to TABLUK, which compares them with well-known dimensionless numbers taken from the Land Table. When a match is found, the name(s) of all numbers in the table fitting the match are printed.

Following the table lookup, a new numerical operator is selected (subroutine PRMTE) and the evaluation and lookup process repeated. When all numerical operators have been processed, a new ordering of the equations is generated (this is a random process performed in ONEMR which also keeps track of which permutations have already been made) and the entire solution process is repeated. Up to 720 solutions are permitted, so all solutions will be generated even for relatively large problems. Sample program outputs appear in Appendix IV.

V. DISCUSSION

In an earlier section, we have seen what program BUCKY does. Namely, it forms solutions to the Buckingham Pi Theorem, evaluates these solutions, and finally, identifies them when possible. In addition, reordering the dimensional equations allows for different solution sets to be formed. When there are few equations to be solved, every possible solution is formed; therefore, an optimal solution is always generated[3,4,5]. Furthermore, numeric substitution into the symbolic form generates scalar values (evaluated solutions) which may be kept in a data base for further use. When an investigator finds a solution particularly suited to his needs, selective retrieval of evaluated solutions allows data to be plotted as well as used in regression, clustering, or factor analysis. As a last step, an attempt is made to identify each solution (primary and algebraically manipulated) by comparing solutions with well-known nondimensional numbers. It should also be noted that by keeping solutions in symbolic form one does not need to refer to other documents to determine which variables are involved in the solution.

There are some shortcomings to program BUCKY. First, no attempt is made to verify that there is a consistent set of equations (none of the other computer programs does this either). Failure to provide a consistent set of equations results in a singular matrix which cannot be inverted. Second, the program does not provide for an automatic change of units[6]. This is correctable by providing conversion factors and

having a separate conversion calculation phase before evaluation occurs. Finally, while no other program attempts to identify solutions, not all well-known solutions are tabled. This can be easily corrected by adding solutions to the table. In addition, some well-known solutions are not recognized because of dimensional substitutions which can be but are not made. For instance,

$$\frac{A^{\frac{1}{2}}\rho V}{\mu}$$

is a Reynold's number; however, the program does not realize that $A^{\frac{1}{2}} = L$ (A is area, L is length). This fault can also be corrected by substituting fundamental units for variables; however, the expense of making all such substitutions does not seem to warrant the gain (there are always numerous little things one can do!)

No further extension of this work is contemplated at the present time.

ACKNOWLEDGEMENT

The author would like to thank Dr. Benajmin E. Cummings for encouraging this work and for providing a meaningful data set for the example used in this report.

REFERENCES

1. Sedov, L. I., Similarity and Dimensional Methods in Mechanics, New York, Academic Press, 1959.

2. Happ, W. W., Private Communication, January 1976.

3. Happ, W. W., "Computer-Oriented Procedures for Dimensional Analysis", Journal of Applied Physics, 38, 3918-3926, September 1967.

4. Sloan, A. D. and Happ, W. W., "Computer Program for Dimensional Analysis", Electronics Research Center, Cambridge, MA, NASA TN D-5165, April 1969.

5. Chen, W. K., "Algebric Theory of Dimensional Analysis", Journal of the Franklin Institute, 292, 403-422, December 1971.

6. Cohen, J. and Ferrari, J. O., "A Conversational Language for Solving Problems in Dimensional Analysis", Computer Methods in Applied Mechanics and Engineering, 5, 53-67, January 1975.

7. Hirschberg, M. A., "A Computer Solution of the Buckingham Pi Theorem Using SYMBOLANG, A Symbol Manipulation Language", USA Ballistic Research Laboratory Report No. 1824, August 1975. (AD #A016901)

8. Land, N. S., "A Compilation of Nondimensional Numbers", Washington, DC, US Government Printing Office, NASA SP-274, 1972.

9. Buckingham, E., "On Physically Similar Systems: Illustrations of the Use of Dimensional Equations", Physical Review, 2, 345-376, 1914.

10. Langhaar, H. L., Dimensional Analysis and Theory of Models, New York, Wiley, 1951.

11. Housner, C. W. and Hudson, D. E., Applied Mechanics Dynamics, New York, van Nostrand, 1950.

12. Findler, N. V., Pfaltz, J. L. and Bernstein, H. J., Four High-Level Extensions of FORTRAN IV, SLIP, AMPPL-II, TREETRAN, SYMBOLANG. New York, Spartan, 305-387, 1972.

13. Hirschberg, M. A., "SYMBOLANG, A SLIP Extension for Algebraic Manipulation",USA Ballistic Research Laboratory Report No. 1749, November 1974. (AD #A003190)

APPENDIX I

PROGRAM COMPUTER LISTING

```
      PROGRAM BUCKY
C
C THIS IS THE PI THEOREM SOLVER
C WRITTEN BY MA HIRSCHBERG
C OCTOBER 1974
C
C SET UP SLIP STORAGE
      COMMON AVSL,X(100)
      COMMON/MAXMIN/XAMAX,XAMIN
      DIMENSION SP(20000)
C SET UP PROGRAM STORAGE
      DIMENSION LNEW(100),LTEMP(100),LPOWER(100)
      DIMENSION LFORMS(100)
      DIMENSION APRIMS(6,100)
      DIMENSION XFRMLA(100),XSOLS(100)
      DIMENSION IVAL(24)
      DIMENSION SOLTN(24)
      DIMENSION IZEE(78)
      DIMENSION FFORM(100)
      DIMENSION LEXP(78)
      DIMENSION EEXPO(5)
      DIMENSION NSOLS(720),IRAYS(20)
      DATA EEXPO/1.,2.,3.,.5,.3333333333/
      DATA IMAXY/7/
C DEFINE EXPONENTS   A-Z,A1-Z1,A2-Z2
      DATA IZEE/1HA,1HB,1HC,1HD,1HE,1HF,1HG,1HH,1HI,1HJ,1HK,1HL,1HM,1HN,
     1 1HO,1HP,1HQ,1HR,1HS,1HT,1HU,1HV,1HW,1HX,1HY,1HZ,
     22HA1,2HB1,2HC1,2HD1,2HE1,2HF1,2HG1,2HH1,2HI1,2HJ1,2HK1,2HL1,2HM1,
     32HN1,2HO1,2HP1,2HQ1,2HR1,2HS1,2HT1,2HU1,2HV1,2HW1,2HX1,2HY1,2HZ1,
     42HA2,2HB2,2HC2,2HD2,2HE2,2HF2,2HG2,2HH2,2HI2,2HJ2,2HK2,2HL2,2HM2,
     52HN2,2HO2,2HP2,2HQ2,2HR2,2HS2,2HT2,2HU2,2HV2,2HW2,2HX2,2HY2,2HZ2/
C
C SETUP WORKING STORAGE
      CALL INITAS(SP,20000)
    2 CONTINUE
C READ NUMBER OF PRIMITIVES
      READ (5,10) NPRMS
   10 FORMAT (I5)
      WRITE (6,12) NPRMS
   12 FORMAT(1H1,55X,18H PI THEOREM SOLVER/
     1 21H NUMBER OF PRIMITIVES,4X,I5)
      WRITE (6,13)
   13 FORMAT (26HOLAND CANDIDATES REFERS TO/
     1 57H A COMPILATION OF DIMENSIONLESS NUMBERS BY NORMAN S. LAND/
     2 48H NASA SP-274, USGPO STOCK NUMBER 3300-0408, 1972)
C THE PRIMITIVES ALSO CONTAIN VALUES FOR DETERMINING THE VALUE OF A PI
C TERM.  IF A VALUE IS -9999.9, THEN NO MORE EVALUATIONS WILL BE MADE WITH
C THAT VARIABLE.
      WRITE (6,14)
   14 FORMAT (30HOPRIMITIVES USED IN EVALUATION)
      WRITE (6,18)
   18 FORMAT (1H ,10HPRIMITIVES,22X,6HVALUES)
C READ PRIMITIVES
      IEVAL=0
      DO 25 I=1,NPRMS
      READ (5,21) (APRIMS(J,I),J=1,6)
```

```
   21 FORMAT (A10,2X,5E12.8)
C
C COUNT THE NUMBER OF VARIABLES FOR EVALUATION
      II=0
      DO 22 J=2,6
      IF (APRIMS(J,I).EQ.-9999.9) GO TO 23
      II=II+1
      IEVAL=1
   22 CONTINUE
   23 CONTINUE
      IVAL(I)=II
C OUTPUT PRIMITIVES
      WRITE (6,24) (APRIMS(J,I),J=1,6)
   24 FORMAT (1H ,A10,2X,5E20.8)
   25 CONTINUE
C READ NUMBER OF FORMULAS
      READ (5,10) NFRMS
C READ NUMBER OF INDEPENDENT VARIABLES
      READ (5,10) IVARS
      WRITE (6,32) NFRMS,IVARS
   32 FORMAT (10HOTHERE ARE,1X,I5,1X,8HFORMULAS,
     1 1X,9HINVOLVING,1X,I5,1X,9HVARIABLES)
      IXTRA=IVARS+1
      CALL COMBY(NFRMS,IXTRA,ICOMS)
      WRITE (6,34) ICOMS
   34 FORMAT (10HOTHERE ARE,1X,I5,1X,21HPOSSIBLE COMBINATIONS)
      ICCC=0
      DO 36 I=1,NFRMS
      IRAYS(I)=I
      ICCC=10*ICCC+I
   36 CONTINUE
      NSOLS(1)=ICCC
      NCT=1
C READ FORMULAS
      DO 40 I=1,NFRMS
      CALL INLIST(LFORMS(I),5HINPUT,3HVAL,TEMP)
   40 CONTINUE
C
C CALCULATE THE NUMBER OF PI TERMS
      IPI=NFRMS-IVARS
      WRITE (6,42) IPI
   42 FORMAT (10HOTHERE ARE,1X,I5,1X,8HPI TERMS)
C
C OUTPUT FORMULAS
C     DO 50 I=1,NFRMS
C     CALL LSQPNT(LFORMS(I),8HFORMULAS,999.,TEMP)
C  50 CONTINUE
C
C STRIP EQUALS OFF AND SETUP NEW SYMBOLANG LISTS
      IGON=0
   55 CONTINUE
      IXDT=0
```

```
      DO 200 I=1,NFRMS
C
C SET UP READER FOR FORMULAS
      III=IRAYS(I)
      LRD=LRDROV(LFORMS(III))
C
C STRIP FIRST PART OF FORMULA
      DO 60 II=1,4
      DATUM=ADVSER(LRD,FLAG)
      IF (II.NE.3) GO TO 60
      IXDT=IXDT+1
C
C SAVE VARIABLE NAME IN FORMULA FOR LATER USE    (SOLUTION)
      FFORM(IXDT)=DATUM
   60 CONTINUE
      LW=0
      LW=LIST(LW)
      IC=0
   65 CONTINUE
C
C ADVANCE THROUGH LIST
      DATUM=ADVSER(LRD,FLAG)
       IF (FLAG.NE.0.) GO TO 70
C
C SET UP TEMPORARY LIST AND COUNT ELEMENTS
      IC=IC+1
      CALL NEWBOT(DATUM,LW,TEMP)
      GO TO 65
   70 CONTINUE
C
C SET COUNT AND FORM NEW NEW SYMBOLANG LIST
      LNEW(I)=0
      LNEW(I)=LIST(LNEW(I))
      LRD=LRDROV(LW)
      LTEMP(I)=LIST(9)
      CALL NEWBOT(LTEMP(I),LNEW(I),TEMP)
      LC=IC-2
      IF (LC.LE.0) GO TO 2000
      DO 80 J=1,LC
      DATUM=ADVSER(LRD,FLAG)
      CALL NEWBOT(DATUM,LTEMP(I),TEMP)
   80 CONTINUE
C
C ERASE TEMPORARY LIST AND PRINT NEW LIST
      CALL IRALST(LW,TEMP)
C     CALL LSQPNT(LNEW(I),4HLNEW,999.,TEMP)
  200 CONTINUE
      IF (IGON.EQ.1) GO TO 220
      IGON=1
      K=NFRMS-1
```

```
      DO 210 I=1,K
C
C SETUP EXPONENTS
      LEXP(I)=LSQMN1(LSQMN3(1.,IZEE(I),1.))
  210 CONTINUE
  220 CONTINUE
C
C SETUP PI TERMS
      LFRMS=LSQMN1(LSQMN1(1.))
      DO 260 I=1,IPI
      LTV=LSQMN1(LSQMN1(1.))
      KK=0
C CYCLE THROUGH FORMULAS MOST IMPORTANT AND SKIP FORMULAS SO AS
C TO INCLUDE ONLY THE IMPORTANT FORMULAS ONE AT A TIME
      DO 245 J=1,NFRMS
      IF (J.EQ.I) GO TO 215
      IF (J+IVARS.LE.NFRMS) GO TO 245
      KK=KK+1
C
C RAISE POWER
      LPOWER(KK)=LSQRAZ(LNEW(J),LEXP(KK))
      LTU=LSQMEX(LTV,LPOWER(KK))
      CALL LSQDES(LTV,TEMP)
      TLGD=SEQRDR(LTU)
      LTV=LSQCPY(TLGD)
      CALL LSQDES(LTU,TEMP)
      GO TO 245
  215 CONTINUE
      CALL LSQDES(LTV,TEMP)
      LTV=LSQMEX(LFRMS,LNEW(J))
  245 CONTINUE
      CALL LSQPNT(LTV,3HLTV,999.,TEMP)
C     CALL PRLSTS(LTV,4)
C
C SOLVE EQUATIONS
      CALL BUCKSV(LTV,IZEE,SOLTN,ICOUNT)
      CALL LSQDES(LTV,TEMP)
      DO 250 J=1,KK
      CALL LSQDES(LPOWER(J),TEMP)
  250 CONTINUE
C
C PRINT SOLUTION FOR PI TERM
      ITIME=1
  251 CONTINUE
      NOM=0
      WRITE (6,252)
  252 FORMAT (45HOSOLUTION OR MANIPULATED SOLUTION FOR PI TERM)
      KK=0
```

```
      DO 258 J=1,NFRMS
      IF (J.NE.I) GO TO 255
      IF (ITIME.LE.5) PTOUT=EEXPO(ITIME)
      IF (ITIME.EQ.6) PTOUT=1./XAMIN
      IF (ITIME.EQ.7) PTOUT=1./XAMAX
      WRITE (6,253) FFORM(I),PTOUT
      NOM=NOM+1
      XFRMLA(NOM)=FFORM(I)
      XSOLS(NOM)=PTOUT
  253 FORMAT (1H ,A10,1X,2H**,1X,E14.8)
  255 CONTINUE
      IF (J+IVARS.LE.NFRMS) GO TO 258
      KK=KK+1
      WRITE (6,253) FFORM(J),SOLTN(KK)
      NOM=NOM+1
      XFRMLA(NOM)=FFORM(J)
      XSOLS(NOM)=SOLTN(KK)
  258 CONTINUE
      IF (ITIME.GE.6 .AND. IFIX(ABS(PTOUT)).EQ.1) GO TO 259
      IF (IEVAL.EQ.1)
     1CALL EVALP(APRIMS,XFRMLA,XSOLS,NOM,NPRMS,IVAL)
      CALL TABLUK(NOM,XSOLS)
  259 CONTINUE
      IF (ITIME.GE.IMAXY) GO TO 260
      CALL PRMTE(ITIME,IVARS,SOLTN)
      ITIME=ITIME+1
      GO TO 251
  260 CONTINUE
      DO 270 I=1,NFRMS
      CALL IRALST(LTEMP(I),TEMP)
      CALL IRALST(LNEW(I),TEMP)
  270 CONTINUE
      CALL ONEMR(NSOLS,IRAYS,NCT,NFRMS,ICOMS)
      IF (NCT.GE.1) GO TO  55
C     GO TO 2
      CALL EXIT
 2000 CONTINUE
      WRITE (6,2010)
 2010 FORMAT (7H NO NO      )
      CALL EXIT
      END
```

```
      SUBROUTINE BUCKSV(LIST, ZEE,SOLTN,ICOUNT)
C
C THIS SUBROUTINE SETS UP THE SOLUTION FOR THE PI THEOREM
C WRITTEN BY MA HIRSCHBERG
C JANUARY 1975
C
      DIMENSION  ZEE(78)
      DIMENSION SOLTN(24)
      DIMENSION AMAT(24,24)
      DIMENSION AWORD(3)
C
C CLEAR STORAGE
      DO 5 I=1,24
      SOLTN(I)=0.0
      DO 4 J=1,24
      AMAT(I,J)=0.0
    4 CONTINUE
    5 CONTINUE
C
C SET UP READER FOR LIST
      LR=LRDROV(LIST)
C SET FLAGS
      LEVEL=0
      ICOUNT=1
      IEND=0
      IWORD=0
      IGO=0
      JEND=1
   10 CONTINUE
      IGO=IGO+1
      JGO=0
C ADVANCE THROUGH LIST
      X=ADVSWR(LR,K)
      IF (K) 100,20,100
   20 IF (LEVEL-LCNTR(LR)) 150,30,70
   30 IF (NAMTST(X)) 60,40,60
   40 IF (LISTMT(X)) 50,10,50
C WE HIT A SUBLIST
   50 CONTINUE
      LEVEL=LEVEL+1
      IEND=0
      IWORD=0
      GO TO 10
C WE HIT A DATUM ELEMENT
   60 CONTINUE
      IEND=0
      IF (IGO.LE.3) GO TO 10
      IF (JGO.EQ.1) ICOUNT=ICOUNT+1
      IP (JGO.EQ.1) GO TO 10
```

```
      IWORD=IWORD+1
C STORE DATUM
      AWORD(IWORD)=X
      GO TO 10
C WE HIT AN END OF SUBLIST
   70 CONTINUE
      LEVEL=LEVEL-1
      IEND=IEND+1
      IF (IEND.LT.2) GO TO 75
      JGO=1
      GO TO 20
   75 CONTINUE
      IF (IWORD.GT.1) GO TO 80
      IF (IWORD.LE.0) GO TO 20
C STORE NUMERICAL COEFFICIENT  (CONSTANT TERM)
      SOLTN(ICOUNT)=-AWORD(1)
      IWORD=0
      GO TO 20
   80 CONTINUE
C STORE MATRIX COEFFICIENT
      DO 90 I=1,78
      IF (AWORD(2).NE. ZEE(I)) GO TO 90
      AMAT(ICOUNT,I)=AWORD(1)
      IWORD=0
      GO TO (20,160), JEND
   90 CONTINUE
   95 CONTINUE
      CALL SLPERR(10H  BUCKSV  )
  100  IF (LEVEL-LCNTR(LR)) 150,120,110
  110 CONTINUE
      LEVEL=LEVEL-1
      GO TO 100
  120 CONTINUE
      CALL RCELL(LR)
  150 CONTINUE
      JEND=2
      GO TO 80
  160 CONTINUE
C
C INVERT MATRIX TO FIND NUMERICAL SOLUTION
      CALL MATINV(AMAT,ICOUNT,SOLTN,24,1,DET)
      IF (DET.EQ.0.) GO TO 95
      RETURN
      END
```

```
      SUBROUTINE COMBY(N,I,IOUT)
C
C THIS ROUTINE CALCULATES THE NUMBER OF COMBINATIONS OF
C N ITEMS TAKEN I AT A TIME
C WRITTEN BY MA HIRSCHBERG
C DECEMBER 1975
C
      IOUT=FACTRL(N)/(FACTRL(N-I)*FACTRL(I))
      RETURN
      END
```

```
      SUBROUTINE EVALP(AP,FFRM,SOL,NOM,NTOT,IVAL)
C
C THIS ROUTINE EVALUATES PI TERMS
C WRITTEN BY MA HIRSCHBERG
C NOVEMBER 1975
C
      COMMON/MAXMIN/XAMAX,XAMIN
      DIMENSION AP(6,100),FFRM(100),SOL(24),IVAL(24)
      DIMENSION EXPNO(24),INDX(24)
      DIMENSION ARAY(24)
C
C AP ARRAY WITH PRIMITIVE NAME AND UP TO 5 VALUES
C FFRM ARRAY WITH NAME OF SOLUTIONS
C SOL ARRAY WITH NUMERIC SOLUTIONS
C NFRMS NUMBER OF FORMULAS
C IVAR NUMBER OF VARIABLES
C IVAL NUMBER OF VALUES FOR EACH VARIABLE
C
C IF AP HAS AN ENTRY WITH THE VALUE -9999.9 END EVALUATION
C
      WRITE (6,10)
   10 FORMAT (27HOVALUE(S) OF PI TERM FOLLOW)
      DO 20 I=1,NOM
      EXPNO(I)=SOL(I)
   20 CONTINUE
      NUSE=6
      I1=NUSE
      I2=NUSE
      I3=NUSE
      I4=NUSE
      I5=NUSE
      I6=NUSE
      I7=NUSE
      I8=NUSE
      I9=NUSE
      I10=NUSE
      I11=NUSE
      I12=NUSE
      I13=NUSE
      I14=NUSE
      I15=NUSE
      I16=NUSE
      I17=NUSE
      I18=NUSE
      I19=NUSE
      I20=NUSE
      I21=NUSE
      I22=NUSE
      I23=NUSE
      I24=NUSE
```

```
ND1=NUSE
ND2=NUSE
ND3=NUSE
ND4=NUSE
ND5=NUSE
ND6=NUSE
ND7=NUSE
ND8=NUSE
ND9=NUSE
ND10=NUSE
ND11=NUSE
ND12=NUSE
ND13=NUSE
ND14=NUSE
ND15=NUSE
ND16=NUSE
ND17=NUSE
ND18=NUSE
ND19=NUSE
ND20=NUSE
ND21=NUSE
ND22=NUSE
ND23=NUSE
ND24=NUSE
TERM1=1.
TERM2=1.
TERM3=1.
TERM4=1.
TERM5=1.
TERM6=1.
TERM7=1.
TERM8=1.
TERM9=1.
TERM10=1.
TERM11=1.
TERM12=1.
TERM13=1.
TERM14=1.
TERM15=1.
TERM16=1.
TERM17=1.
TERM18=1.
TERM19=1.
TERM20=1.
TERM21=1.
TERM22=1.
TERM23=1.
TERM24=1.
IUSE=0
```

```
      DO 50 J=1,NOM
      FR=FFRM(J)
      DO 40 K=1,NTOT
      AUSE=AP(1,K)
      IF (FR.NE.AUSE) GO TO 40
      IUSE=IUSE+1
      INDX(IUSE)=K
      GO TO 50
   40 CONTINUE
      WRITE (6,42) FR
   42 FORMAT (15HONAME NOT FOUND,5X,A10)
      CALL EXIT
   50 CONTINUE
      KUSE=25-NOM
      DO 150 K=1,NOM
      LL=NOM-K+1
      IUSE=INDX(LL)
      II=IVAL(IUSE)
      IF (II.EQ.0) GO TO 130
      GO TO (124,123,122,121,120,119,118,117,116,115,114,113,112,111,
     1 110,109,108,107,106,105,104,103,102,101), KUSE
  101 I1=II
      ND1=1
      GO TO 130
  102 I2=II
      ND2=1
      GO TO 130
  103 I3=II
      ND3=1
      GO TO 130
  104 I4=II
      ND4=1
      GO TO 130
  105 I5=II
      ND5=1
      GO TO 130
  106 I6=II
      ND6=1
      GO TO 130
  107 I7=II
      ND7=1
      GO TO 130
  108 I8=II
      ND8=1
      GO TO 130
  109 I9=II
      ND9=1
      GO TO 130
  110 I10=II
      ND10=1
      GO TO 130
```

```
111 I11=II
    ND11=1
    GO TO 130
112 I12=II
    ND12=1
    GO TO 130
113 I13=II
    ND13=1
    GO TO 130
114 I14=II
    ND14=1
    GO TO 130
115 I15=II
    ND15=1
    GO TO 130
116 I16=II
    ND16=1
    GO TO 130
117 I17=II
    ND17=1
    GO TO 130
118 I18=II
    ND18=1
    GO TO 130
119 I19=II
    ND19=1
    GO TO 130
120 I20=II
    ND20=1
    GO TO 130
121 I21=II
    ND21=1
    GO TO 130
122 I22=II
    ND22=1
    GO TO 130
123 I23=II
    ND23=1
    GO TO 130
124 I24=II
    ND24=1
130 CONTINUE
    KUSE=KUSE+1
150 CONTINUE
    OLDTRM=-9999.9
    DO 250 LL=1,1
    DO 248 J1=1,I1,ND1
    IF (I1.EQ.NUSE) GO TO 240
    CALL NUMB(KK,INDX,1,IUSE,KUSE)
    ARAY(1)=AP(J1+1,IUSE)
```

```
TERM1=ARAY(1)**EXPNO(KUSE)
DO 248 J2=1,I2,ND2
IF (I2.EQ.NUSE) GO TO 240
CALL NUMB(KK,INDX,2,IUSE,KUSE)
ARAY(2)=AP(J2+1,IUSE)
TERM2=ARAY(2)**EXPNO(KUSE)
DO 248 J3=1,I3,ND3
IF (I3.EQ.NUSE) GO TO 240
CALL NUMB(KK,INDX,3,IUSE,KUSE)
ARAY(3)=AP(J3+1,IUSE)
TERM3=ARAY(3)**EXPNO(KUSE)
DO 248 J4=1,I4,ND4
IF (I4.EQ.NUSE) GO TO 240
CALL NUMB(KK,INDX,4,IUSE,KUSE)
ARAY(4)=AP(J4+1,IUSE)
TERM4=ARAY(4)**EXPNO(KUSE)
DO 248 J5=1,I5,ND5
IF (I5.EQ.NUSE) GO TO 240
CALL NUMB(KK,INDX,5,IUSE,KUSE)
ARAY(5)=AP(J5+1,IUSE)
TERM5=ARAY(5)**EXPNO(KUSE)
DO 248 J6=1,I6,ND6
IF (I6.EQ.NUSE) GO TO 240
CALL NUMB(KK,INDX,6,IUSE,KUSE)
ARAY(6)=AP(J6+1,IUSE)
TERM6=ARAY(6)**EXPNO(KUSE)
DO 248 J7=1,I7,ND7
IF (I7.EQ.NUSE) GO TO 240
CALL NUMB(KK,INDX,7,IUSE,KUSE)
ARAY(7)=AP(J7+1,IUSE)
TERM7=ARAY(7)**EXPNO(KUSE)
DO 248 J8=1,I8,ND8
IF (I8.EQ.NUSE) GO TO 240
CALL NUMB(KK,INDX,8,IUSE,KUSE)
ARAY(8)=AP(J8+1,IUSE)
TERM8=ARAY(8)**EXPNO(KUSE)
DO 248 J9=1,I9,ND9
IF (I9.EQ.NUSE) GO TO 240
CALL NUMB(KK,INDX,9,IUSE,KUSE)
ARAY(9)=AP(J9+1,IUSE)
TERM9=ARAY(9)**EXPNO(KUSE)
DO 248 J10=1,I10,ND10
IF (I10.EQ.NUSE) GO TO 240
CALL NUMB(KK,INDX,10,IUSE,KUSE)
ARAY(10)=AP(J10+1,IUSE)
TERM10=ARAY(10)**EXPNO(KUSE)
DO 248 J11=1,I11,ND11
IF (I11.EQ.NUSE) GO TO 240
CALL NUMB(KK,INDX,11,IUSE,KUSE)
ARAY(11)=AP(J11+1,IUSE)
TERM11=ARAY(11)**EXPNO(KUSE)
```

```
DO 248 J12=1,I12,ND12
IF (I12.EQ.NUSE) GO TO 240
CALL NUMB(KK,INDX,12,IUSE,KUSE)
ARAY(12)=AP(J12+1,IUSE)
TERM12=ARAY(12)**EXPNO(KUSE)
DO 248 J13=1,I13,ND13
IF (I13.EQ.NUSE) GO TO 240
CALL NUMB(KK,INDX,13,IUSE,KUSE)
ARAY(13)=AP(J13+1,IUSE)
TERM13=ARAY(13)**EXPNO(KUSE)
DO 248 J14=1,I14,ND14
IF (I14.EQ.NUSE) GO TO 240
CALL NUMB(KK,INDX,14,IUSE,KUSE)
ARAY(14)=AP(J14+1,IUSE)
TERM14=ARAY(14)**EXPNO(KUSE)
DO 248 J15=1,I15,ND15
IF (I15.EQ.NUSE) GO TO 240
CALL NUMB(KK,INDX,15,IUSE,KUSE)
ARAY(15)=AP(J15+1,IUSE)
TERM15=ARAY(15)**EXPNO(KUSE)
DO 248 J16=1,I16,ND16
IF (I16.EQ.NUSE) GO TO 240
CALL NUMB(KK,INDX,16,IUSE,KUSE)
ARAY(16)=AP(J16+1,IUSE)
TERM16=ARAY(16)**EXPNO(KUSE)
DO 248 J17=1,I17,ND17
IF (I17.EQ.NUSE) GO TO 240
CALL NUMB(KK,INDX,17,IUSE,KUSE)
ARAY(17)=AP(J17+1,IUSE)
TERM17=ARAY(17)**EXPNO(KUSE)
DO 248 J18=1,I18,ND18
IF (I18.EQ.NUSE) GO TO 240
CALL NUMB(KK,INDX,18,IUSE,KUSE)
ARAY(18)=AP(J18+1,IUSE)
TERM18=ARAY(18)**EXPNO(KUSE)
DO 248 J19=1,I19,ND19
IF (I19.EQ.NUSE) GO TO 240
CALL NUMB(KK,INDX,19,IUSE,KUSE)
ARAY(19)=AP(J19+1,IUSE)
TERM19=ARAY(19)**EXPNO(KUSE)
DO 248 J20=1,I20,ND20
IF (I20.EQ.NUSE) GO TO 240
CALL NUMB(KK,INDX,20,IUSE,KUSE)
ARAY(20)=AP(J20+1,IUSE)
TERM20=ARAY(20)**EXPNO(KUSE)
DO 248 J21=1,I21,ND21
IF (I21.EQ.NUSE) GO TO 240
CALL NUMB(KK,INDX,21,IUSE,KUSE)
ARAY(21)=AP(J21+1,IUSE)
TERM21=ARAY(21)**EXPNO(KUSE)
```

```
      DO 248 J22=1,I22,ND22
      IF (I22.EQ.NUSE) GO TO 240
      CALL NUMB(KK,INDX,22,IUSE,KUSE)
      ARAY(22)=AP(J22+1,IUSE)
      TERM22=ARAY(22)**EXPNO(KUSE)
      DO 248 J23=1,I23,ND23
      IF (I23.EQ.NUSE) GO TO 240
      CALL NUMB(KK,INDX,23,IUSE,KUSE)
      ARAY(23)=AP(J23+1,IUSE)
      TERM23=ARAY(23)**EXPNO(KUSE)
      DO 248 J24=1,I24,ND24
      IF (I24.EQ.NUSE) GO TO 240
      CALL NUMB(KK,INDX,24,IUSE,KUSE)
      ARAY(24)=AP(J24+1,IUSE)
      TERM24=ARAY(24)**EXPNO(KUSE)
  240 CONTINUE
      TERMO=     TERM1*TERM2*TERM3*TERM4*TERM5*TERM6*TERM7*TERM8
     1 *TERM9*TERM10*TERM11*TERM12*TERM13*TERM14*TERM15*TERM16
     2 *TERM17*TERM18*TERM19*TERM20*TERM21*TERM22*TERM23*TERM24
      IF (OLDTRM.EQ.TERMO) GO TO 246
      WRITE (6,241) (ARAY(III),III=1,NOM)
  241 FORMAT (34H0VALUES USED IN EVALUATING PI TERM/
     1 (1H ,6E20.8))
      RECIP=1./TERMO
      WRITE (6,245) TERMO,RECIP
  245 FORMAT (18H0EVALUATED TERM = ,E20.8,2X,13HRECIPROCAL = ,E20.8)
  246 CONTINUE
      OLDTRM=TERMO
  248 CONTINUE
  250 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE NUMB(KK,INDX,LOOP,IUSE,KUSE)
C
C WRITTEN BY MA HIRSCHBERG
C NOVEMBER 1975
C
      DIMENSION INDX(24)
      KUSE=LOOP
      IUSE=INDX(KUSE)
      RETURN
      END
```

```
      SUBROUTINE ONEMR(NSOLS,IRAYS,NCT,IXTRA,IBYPS)
C
C THIS ROUTINE PERMUTES THE INPUT EQUATIONS
C WRITTEN BY MA HIRSCHBERG
C DECEMBER 1975
C
      DIMENSION NSOLS(720),IRAYS(20)
      DIMENSION IUSE(20)
      DATA IOK/0/
C
C NSOLS        ARRAY SHOWING UNIQUENESS OF PERMUTATION
C IRAYS        ARRAY WITH NEXT PERMUTATION
C NCT          NUMBER OF PERMUTATIONS
C IXTRA        NUMBER OF ITEMS IN ARRAY IRAYS
C IBYPS        TOTAL NUMBER OF PERMUTATIONS
C
      IF (NCT+1.GT.720) NCT=-1
      IF (NCT+1.GT.IBYPS) NCT=-1
      IF (NCT.EQ.-1) RETURN
      ITIMES=0
      XTRA=IXTRA
   10 CONTINUE
      ITIMES=ITIMES+1
      IF (ITIMES.LE.1000) GO TO 15
      NCT=-1
      RETURN
   15 CONTINUE
      DO 20 I=1,IXTRA
      IUSE(I)=I
   20 CONTINUE
      DO 50 I=1,IXTRA
   30 CONTINUE
      XUSE=URAN31(IOK)*XTRA+1.
      IF (XUSE.GT.XTRA) XUSE=XTRA
      IPUT=XUSE
      IF (IUSE(IPUT).LE.0) GO TO 30
      IRAYS(I)=IPUT
      IUSE(IPUT)=0
   50 CONTINUE
      ITEST=0
      DO 60 I=1,IXTRA
      ITEST=10*ITEST+IRAYS(I)
   60 CONTINUE
      DO 70 I=1,NCT
      IF (ITEST.EQ.NSOLS(I)) GO TO 10
   70 CONTINUE
      NCT=NCT+1
      NSOLS(NCT)=ITEST
C
      RETURN
      END
```

```
      SUBROUTINE PRMTE(ITIME,NTOT,SOL)
C
C THIS ROUTINE MANIPULATES PI SOLUTIONS
C WRITTEN BY MA HIRSCHBERG
C NOVEMBER 1975
C
      COMMON/MAXMIN/XAMAX,XAMIN
      COMMON/SOLSAV/SOL1
      DIMENSION SOL(25)
      DIMENSION SOL1(25)
C
      GO TO (10,30,50,70,90,110), ITIME
C
C SQUARE NUMBERS
   10 CONTINUE
      DO 20 I=1,NTOT
      SOL1(I)=SOL(I)
      SOL(I)=2.*SOL(I)
   20 CONTINUE
      RETURN
C
C CUBE NUMBERS
   30 CONTINUE
      DO 40 I=1,NTOT
      SOL(I)=3.*SOL1(I)
   40 CONTINUE
      RETURN
C
C SQUARE ROOT
   50 CONTINUE
      DO 60 I=1,NTOT
      SOL(I)=.5*SOL1(I)

   60 CONTINUE
      RETURN
C
C CUBE ROOT
   70 CONTINUE
      A=1./3.
      DO 80 I=1,NTOT
      SOL(I)=A*SOL1(I)
   80 CONTINUE
      RETURN
C
C FIND MAX AND MIN
   90 CONTINUE
      XAMAX=-10000.
      XAMIN=10000.
      DO 95 I=1,NTOT
```

```
      IF (SOL1(I).EQ.0.0) GO TO 95
      XAMAX=AMAX1(XAMAX,ABSF(SOL1(I)))
      XAMIN=AMIN1(XAMIN,ABSF(SOL1(I)))
   95 CONTINUE
      DO 100 I=1,NTOT
      SOL(I)=SOL1(I)/XAMIN
  100 CONTINUE
      RETURN
  110 CONTINUE
      DO 120 I=1,NTOT
      SOL(I)=SOL1(I)/XAMAX
  120 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE TABLUK(NTOT,SOLTN)
C
C WRITTEN BY MA HIRSCHBERG
C NOVEMBER 1975
C
C THIS PROGRAM COMPARES PI THEOREM SOLUTIONS WITH LAND NUMBERS
C
      DIMENSION SOLTN(24)
      DIMENSION SOL(24)
      DIMENSION ITAB(93)
      DIMENSION ATABL(516)
      DATA ITAB/ 2,4,10,2,2,1,3,15,10,13,2,1,5,1,2,2,1,2,1,1,1,1,1,
     1 4,28,18,6,3,6,4,3,1,1,2,2,1,1,2,1,1,1,
     11,1,1,1,1,1,1,1,1,1,1,1,5,17,2,3,2,1,2,1,3,1,2,1,2,1,1,1,1,1,1,
     1 6, 8,1,2,1,1,1,1,1,1,      7,4,1,1,1,1, 8,3,1,1,1/
      DATA (ATABL(I),I=1,133)/
     *                             -1.,1.,6HCROCCO,5HDEBYE,8HEINSTEIN,
     *7HKNUDSEN,5HLAVAL,4HMACH,4HNAZE,6HSARRAU,10HSMOLUCKOWS,
     *7HSP HEAT, -2.,2.,6HCAUCHY,5HHOOKE, -1.,2.,4HEVAP,4HELAS, 1.,1.,
     *4HHALL,-1.,-1.,1.,9HARRHENIUS,4HEVAP,5HJACOB,7HPRANDTL,5HROSBY,
     *7HRUSSELL,7HSCHMIDT,8HSURF VEL,8HVISOELAS,7HCOLBURN,-1.,1.,1.,
     *4HBIOT,10HBODENSTEIN,8HREYNOLDS,3HCAP,3HNN1,7HNUSSELT,6HPECLET,
     *6HPOSNOV,7HPRANDTL,8HSHERWOOD,8HSTROUHAL,7HTHOMSON,5HTRUNC,-.5,
     *-.5,1.,10HBOUSSINESQ,6HFROUDE,-2.,2.,2.,3HCAP,-2.,-1.,1.,5HEKMAN,
     *4HELAS,5HEULER,7HFANNING,3HNN2,-2.,2.,4.,6HTAYLOR,-1.,-1.,2.,
     *6HECKERT,6HFROUDE,-3.,-1.,1.,4HFLOW,10HMASS RATIO,1.,1.,1.,
     *9HM PRANDTL,-2.,1.,1.,5HREECH,7HFOURIER,-.5,.5,1.,7HPRANDTL,-1.,
     *1.,4.,8HRAD PRES,-.3333333333,.3333333333,1.,5HSACHS,-2.,1.,3.,
     *7HGALILEO,-.6666666666,.3333333333,.3333333333,10HKIRPITCHEF,
     *-1.,-1.,1.,1.,6HBANSEN,7HBINGHAM,7HBOUGUER,7HNUSSELT,8HHEDSTROM,
     *9HKIRPICHEV,9HKOSSOVICH,8HLAGRANGE,10HCRISPATION,9HDAMKOHLER,
     *4HELAS,5HELLIS,6HGRAETZ,8HMOMENTUM,7HNUSSELT,8HPARTICLE,4HPLAS/
      DATA (ATABL(I),I=134,310)/
     *8HPIPELINE,-1.,-1.,1.,2.,9HBRINKMANN,9HDAMKOHLER,10HPOISEUILLE,
     *10HPOMERANTSE,10HPREDVODITL,10HSOMMERFELD,-1.,-1.,-1.,1.,6HHERSEY,
     *8HELSASSER,7HSTANTON,-1.,1.,1.,1.,9HDAMKOHLER,5HLEWIS,6HMERKEL,
     *8HREYNOLDS,10HRICHARDSON,7HSEMENOV,-1.,1.,1.,2.,4HBOND,5HWEBER,
     *6HEOTVOS,9HMAG INTER,-1.,.5,.5,1.,6HALFVEN,6HKARMAN,8HMAG MACA,
     *-.5,.5,1.,1.,8HHARTMANN,-2.,-2.,-1.,3.,3HACC,-3.,-1.,1.,4.,
     *6HMORTON,7HCAP-BOY,-3.,-2.,-1.,1.,8HLAGRANGE,10HHEAT TRANS,-2.,1.,
     *1.,2.,8HHEDSTROM,-1.,-.5,-.5,1.,7HCOWLING,-.5,.5,.5,1.,7HGOUCHER,
     *8HDERYAGIN,-1.,-.5,-.5,.5,5HEKMAN,-2.,-1.,1.,1.,4HELAS,-.4,-.2,.2,
     *1.,5HEXPLO,-2.,1.,1.,1.,8HSURATMAN,-1.5,-.5,.5,1.,7HSLUSH T,-.75,
     *-.75,.5,1.,8HSP SPEED,-3.,-1.,-1.,1.,4HTOMS,-1.,-.5,.5,1.,
     *8HLEVERETT,-2.,-1.,1.,2.,8HMAG PRES,1.,1.,1.,1.,10HM REYNOLDS,
     *-2.,-2.,-1.,1.,6HNEWTON,-.6666666666,.3333333333,.6666666666,1.,
     *7HNUSSELT,-.5,-.5,-.5,1.,9HOHNESORGE,-5.,-3.,-1.,1.,5HPOWER,-1.,
     *.5,1.,1.,6HREGIER,-1.,-1.,-1.,1.,1.,9HKIRPICHEV,8HREYNOLDS,-1.,
     *-1.,1.,1.,1.,7HHODGSON,7HGRAVITY,3HNN5,-2.,-1.,1.,1.,1.,5HJOULE/
```

```
      DATA (ATABL(I),I=311,516)/
     *5HDARCY,-2.,-1.,1.,1.,2.,7HSQUEEZE,-2.,-1.,-1.,1.,1.,6HSTOKES,
     *7HFOURIER,-1.,-1.,-1.,1.,2.,9HMARANGONI,-1.,-1.,1.,1.,2.,
     *7HMAG DYN,7HJEFFREY,9HDAMKOHLER,-.5,1.,1.,1.,1.5,9HLUNDQUIST,-2.,
     *-1.,1.,1.,3.,6HKARMAN,10HARCHIMEDES,-1.,-1.,-1.,.5,1.,3HCAP,
     *-1.,-.666666666,-.333333333,.666666666,1.,8HJ FACTOR,6HCONDEN,
     *-1.,1.,1.,2.,2.,10HCENTRIFUGE,-1.,-1.,1.,1.,3.,8HCLAUSIUS,
     *-1.,-.666666666,.333333333,.666666666,1.,8HJ FACTOR,-1.,-.5,1.,1.,
     *1.5,4HDEAN,-1.,-.5,.5,1.,1.,5HFRUEH,-1.,1.,1.,1.,1.,6HPECLET,
     *-1.,-1.,-1.,1.,1.,2.,7HBAGNOLD,-3.,-1.,-1.,1.,1.,1.,6HTHRING,
     *9HBOLTZMANN,-1.,-1.,-1.,1.,1.,4.,6HSTEFAN,-1.,-1.,-.5,.5,1.,2.,
     *10HELECTROVIS,-1.,-1.,1.,1.,2.,2.,9HMAG FORCE,-1.,-1.,-.5,1.,1.,
     *1.,3HNN3,-1.,-1.,-.5,-.5,1.,1.,3HNN4,-2.,1.,1.,1.,2.,3.,7HGRASHOF,
     *-1.,-1.,-1.,1.,1.,2.,3.,10HCONDENSATI,-1.,-1.,-1.,-1.,1.,1.,1.,
     *9HDAMKOHLER,-2.,-2.,-1.,-1.,1.,2.,2.,6HOCVIRK,-1.,-1.,-1.,1.,1.,
     *1.,2.,8HBUOYANCY,-1.,-1.,1.,1.,1.,1.,2.,3.,8HRAYLEIGH,-3.,-2.,-1.,
     *-1.,1.,1.,1.,4.,7HMCADAMS,-1.,-.5,-.5,-.5,.5,1.,2.,2.,8HLYKOUDIS/
C
C SAVE SOLUTIONS
      IGO=1
      DO 10 I=1,NTOT
      SOL(I)=SOLTN(I)
   10 CONTINUE
C
C SORT ARRAY
   15 CONTINUE
      I=2
   18 CONTINUE
      IF (SOL(I).GE.SOL(I-1)) GO TO 20
      X=SOL(I-1)
      SOL(I-1)=SOL(I)
      SOL(I)=X
      GO TO 15
   20 CONTINUE
      IF (I.EQ.NTOT) GO TO 25
      I=I+1
      GO TO 18
C
C FIND TABLED VALUES FOR SORTED EXPONENTS
   25 CONTINUE
C DO LOOK-UP
      I=1
      K=1
   30 CONTINUE
      IF (I.GT.93) GO TO 300
      IUSE=ITAB(I)
      IF (IUSE.EQ.NTOT) GO TO 50
      I=I+1
      JMUL=ITAB(I)
      K=IUSE*JMUL+K
```

```
      DO 40 II=1,JMUL
      I=I+1
      K=K+ITAB(I)
   40 CONTINUE
      I=I+1
      GO TO 30
C
C WE HAVE THE RIGHT NUMBER OF ELEMENTS
   50 CONTINUE
      KTEMP=K
      ITEMP=I+1
      IDX=ITAB(ITEMP)
      DO 200 II=1,IDX
      ITEMP=ITEMP+1
      LUSE=ITAB(ITEMP)
      DO 100 JJ=1,IUSE
      KUSE=KTEMP+JJ-1
      IF (ABSF(SOL(JJ)-ATABL(KUSE)).LE..0001) GO TO 100
      GO TO 150
  100 CONTINUE
      WRITE (6,110)
  110 FORMAT (16HOLAND CANDIDATES)
      L1=KUSE+1
      L2=L1+LUSE-1
      WRITE (6,120) (ATABL(IJ),IJ=L1,L2)
  120 FORMAT (1H ,8A12)
      GO TO 210
  150 CONTINUE
      KTEMP=KTEMP+IUSE+LUSE
  200 CONTINUE
  210 CONTINUE
C
C INVERT SOLUTION AND TRY AGAIN
      GO TO (260,300), IGO
  260 CONTINUE
      IGO=2
      DO 270 II=1,NTOT
      SOL(II)=-SOLTN(II)
  270 CONTINUE
      GO TO 15
  300 CONTINUE
      RETURN
      END
```

APPENDIX II

SAMPLE PROGRAM INPUTS

NUMERICAL INPUTS

VARIABLE	ENGLISH UNITS	METRIC UNITS
Force (F)	10^4 Lbs	444820×10^4 Dynes
Velocity (V)	10^3 Ft/Sec	$30480 \frac{cm}{sec}$
Area (A)	$10^2 Ft^2$	$92903.04\ cm^2$
Density (δ)	$.00237 \frac{Lb\ sec}{Ft^4}$	$.0012144 \frac{Dyne\ Sec}{cm^4}$
Viscosity (μ)	$4 \times 10^{-7} \frac{Lb\ sec}{Ft^2}$	$1.915201 \frac{Dyne\ Sec}{cm^2}$

PROGRAM INPUTS

```
     8
FORCED       444820.     + 04-9999.9
VISCOSITY    1.915201    - 04-9999.9
AREA          92903.04        -9999.9
DENSITY      .00122144        -9999.9
VELOCITY      30480.          -9999.9
FORCE        -9999.9
LENGTH       -9999.9
TIME         -9999.9
     5
     3
        FORCED=FORCE$$
        VISCOSITY=FORCE*TIME/LENGTH**2$$
        AREA=LENGTH**2$$
        DENSITY-FORCE*TIME**2/LENGTH**4$$
        VELOCITY=LENGTH/TIME$$
NOMOR
$$
```

APPENDIX III

INPUT DESCRIPTION AND FORMAT

CARD TYPE	DATUM NUMBER	NAME OF DATUM	DATA ENTRIES	DATA FORMAT TYPE
1	1	NPRMS	Total number of variables and fundamental units.	1
2	1	APRIMS (1,N)	Name of variable or fundamental unit (there are NPRM Type 2 cards read).	2
2	2-6	APRIMS (2-6,N)	Values for fundamental units (used by the evaluation routine EVALP). -9999.9 signifies end of data. Up to five values are allowed per variable. All combinations of input values are calculated.	
3	1	NFRMS	Number of equations	1
4	1	IVARS	Number of fundamental units	1
5		LFORMS	Dimensional equations (there are NFRMS equations which are read by the SYMBOLANG routine INLIST.	*

FORMATS

Type 1	Format (I5)
Type 2	Format (A10, 2X, 5E12.8)

*Dimensional equations are read by the SYMBOLANG routine INLIST as modified to run on the BRLESC II Computer). See Appendix II for a sample of inputs. Each equation is terminated by a "$$". In addition, a card with "NOMOR" (beginning in card column 1) and a card with "$$" (also beginning in column 1) are used to terminate the INLIST read and terminate input to program BUCKY.

APPENDIX IV

SAMPLE PROGRAM OUTPUTS

```
NUMBER OF PRIMITIVES        8

LAND CANDIDATES REFERS TO
A COMPILATION OF DIMENSIONLESS NUMBERS BY NORMAN S. LAND
NASA SP-274, USGPO STOCK NUMBER 3300-0408, 1972

PRIMITIVES USED IN EVALUATION
PRIMITIVES                        VALUES
FORCED           0.44482000E 10    -0.99999000E 04     0.00000000E 00
VISCOSITY        0.19152010E-03    -0.99999000E 04     0.00000000E 00
AREA             0.92903040E 05    -0.99999000E 04     0.00000000E 00
DENSITY          0.12214400E-02    -0.99999000E 04     0.00000000E 00
VELOCITY         0.30480000E 05    -0.99999000E 04     0.00000000E 00
FORCE           -0.99999000E 04     0.00000000E 00     0.00000000E 00
LENGTH          -0.99999000E 04     0.00000000E 00     0.00000000E 00
TIME            -0.99999000E 04     0.00000000E 00     0.00000000E 00

THERE ARE     5 FORMULAS INVOLVING     3 VARIABLES

THERE ARE     5 POSSIBLE COMBINATIONS
       FORCED=FORCE$$
       VISCOSITY=FORCE*TIME/LENGTH**2$$
       AREA=LENGTH**2$$
       DENSITY=FORCE*TIME**2/LENGTH**4$$
       VELOCITY=LENGTH/TIME$$

THERE ARE     2 PI TERMS
      LTV = FORCB**(1    +    B)*LENGTH**(2*A    -    4*B    +    C)
         *TIME**(2*B    -    C)
 $
 $END OF EXPRESSION

SOLUTION OR MANIPULATED SOLUTION FOR PI TERM
FORCED       **  .10000000E 01
AREA         ** -.10000000E 01
DENSITY      ** -.10000000E 01
VELOCITY     ** -.20000000E 01
```

```
VALUE(S) OF PI TERM FOLLOW

VALUES USED IN EVALUATING PI TERM
     0.44482000E 10      0.92903040E 05      0.12214400E-02      0.30480000E 05

EVALUATED TERM =       0.42194157E-01  RECIPROCAL =       0.23699964E 02

LAND CANDIDATES
  BOND        WEBER        EOTVOS        MAG INTER
```

DISTRIBUTION LIST

No. of Copies	Organization
12	Commander Defense Documentation Center ATTN: DDC-TCA Cameron Station Alexandria, VA 22314
1	Commander US Army Materiel Development and Readiness Command ATTN: DRCDMD-ST 5001 Eisenhower Avenue Alexandria, VA 22333
1	Commander US Army Aviation Research and Development Command ATTN: DRSAV-E 12th and Spruce Streets St. Louis, MO 63166
1	Director US Army Air Mobility Research and Development Laboratory Ames Research Center Moffett Field, CA 94035
2	Commander US Army Electronics Research and Development Command Technical Support Activity ATTN: DELSD-L Fort Monmouth, NJ 07703
1	Commander US Army Communications Rsch and Development Command ATTN: DRDCO-SGS Fort Monmouth, NJ 07703
1	Commander US Army Missile Research and Development Command ATTN: DRDMI-R Redstone Arsenal, AL 35809
1	Commander US Army Missile Materiel Readiness Command ATTN: DRSMI-AOM Redstone Arsenal, AL 35809
1	Commander US Army Tank Automotive Research & Development Cmd ATTN: DRDTA-UL Warren, MI 48090
2	Commander US Army Armament Research and Development Command ATTN: DRDAR-TSS Dover, NJ 07801
1	Commander US Army Armament Research and Development Command ATTN: DRDAR-LC, Dr. Frasier Dover, NJ 07801
1	Commander US Army Armament Research and Development Command ATTN: DRDAR-MS, Mr. Grobstein Dover, NJ 07801
1	Commander US Army Armament Research and Development Command ATTN: DRDAR-SC, S. Goodman Dover, NJ 07801
1	Commander US Army Armament Materiel Readiness Command ATTN: DRSAR-LEP-L, Tech Lib Rock Island, IL 61299

DISTRIBUTION LIST

No. of Copies	Organization
1	Director US Army TRADOC Systems Analysis Activity ATTN: ATAA-SL, Tech Lib White Sands Missile Range NM 88002
1	ASD (Dr. R. Sylvester) Wright-Patterson AFB, OH 45433
1	Office of Technology Assessment US Congress ATTN: Mr. Coates Washington, DC 20510
1	Brookhaven National Labs ATTN: Dr. Herbert J. Bernstein Chemical Dept, Bldg. 555A Upton, Long Island, NY 11973
1	Clippinger Research Labs Ohio State University ATTN: Dr. W. K. Chen Athens, OH 45701
1	Gulf Universities Research Consortium ATTN: Dr. J. Sharp 16821 Buccaneer Lane, Suite 206 Clear Lake City Houston, TX 77058
1	Operating Systems, Inc. ATTN: Dr. C. Montgomery 21031 Ventura Boulevard Woodland Hills, CA 91364
1	Pritsker and Associates, Inc. ATTN: Dr. A. Pritsker 1710 South Street Lafayette, IN 47904
1	Brandeis University Physics Department ATTN: Dr. Jacques Cohen Waltham, MA 02154
1	California State University, Sacramento Dept of Electrical Engineering ATTN: Prof. W. W. Happ 6000 J Street Sacramento, CA 95819
1	IIT Research Institute Engineering Mechanics Division ATTN: Mr. L. Barbarek 10 West 35th Street Chicago, IL 60616
1	Massachusetts Institute of Tech ATTN: Prof. J. Weizenbaum Cambridge, MA 02139
3	Southwest Research Institute ATTN: Dr. P. S. Westine Dr. W. E. Baker Dr. F. T. Dodge 8500 Culebra Road P. O. Drawer 28510 San Antonio, TX 78284
1	State Univ of New York, Buffalo Dept of Computer Science ATTN: Dr. Nicholas V. Findler 4226 Ridge Lea Road Amherst, NY 14226

Aberdeen Proving Ground

Dir, USAMSAA
ATTN: Dr. Sperrazza
Mr. Bain
Mr. Maki
Dr. Cummings
Dr. Dubin
Mr. Belbot
Mr. Bailey
Mr. Reeches

(continued)

DISTRIBUTION LIST

Aberdeen Proving Ground

Cdr, USATECOM
  ATTN: DRSTE-SG-H

Cdr/Dir, CSL, EA
  ATTN: DRDAR-MSE, Mr. Goldberg
  DRDAR-CL
    Dr. Sacco
    Mr. Sturdivan